Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/4B

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs

1(a)	Mean = 504	B1	1.1b
	1.96	B1	3.3
	$504 \pm \frac{5.4}{\sqrt{8}} \times "1.96"$	M1	2.1
	(500.258, 507.742)	A1	1.1b
		(4)	
(b)	**505** is in the confidence **interval** therefore there is evidence that the machine is **working** properly	B1ft	2.2b
		(1)	
(c)	5% oe	B1	1.1b
		(1)	
(d)	s needs to be used instead of σ **and** a t-value instead of the z value	B1	3.3
	since the sample is small therefore you can't use the normal distribution	B1	3.5b
		(2)	
			(8 marks)

Notes:

(a)	B1	504 may be seen in part(b)
	B1	For realising a normal distribution must be used as a model and finding the correct value 1.96
	M1	For $504 \pm \frac{5.4}{\sqrt{8}} \times "z \text{ value}"$. $\lvert z\rvert > 1$ May be implied by a correct CI
	A1	awrt 500.26 and 507.74 NB using t gives 500.29 and 507.71
(b)	B1ft	Drawing a correct inference (ft) using their answer to part (a) and the 505 from the question. Reason must be given. Ignore incorrect non – contextual
(c)	B1	5%
(d)	B1	create new model by using s and t. Allow if state use CI $\mu \pm \frac{s}{\sqrt{n}} \times "t"$ or use $s = 4.44$ and $t = 2.365$
	B1	For recognising that the sample is small

Question	Scheme	Marks	AOs
2(a)	$S_{ww} = 13447 - \frac{303^2}{8} = 1970.875$		
	$r = \frac{269.5}{\sqrt{42 \times 1970.875}}$	M1	1.1b
	$r = 0.9367...$ awrt 0.937	A1	1.1b
		(2)	
(b)	As the amount of **fertiliser** increases the **yield** increases	B1	3.2a
		(1)	
(c)	$b = \frac{269.5}{42}$ $[= 6.41666...]$	M1	3.3
	$a = \frac{303}{8} - 'b'\frac{28}{8}$ $[= 15.41666...]$	M1	1.1b
	$w = 15.4 + 6.42f$	A1	1.1b
		(3)	
(d)	3.21 tonnes	B1ft	1.1b
		(1)	
(e)	The **residual** plot is close to an 'n' shape or the <u>**residuals**</u> appear not to be **randomly** scattered	M1	2.4
	The model in part(c) is unlikely to be suitable	A1	2.2b
		(2)	
(f)	Fit a curve rather than a line	B1	3.5c
		(1)	
			(10 marks)

Notes:

(a)	M1 A1	Complete correct method for finding r for awrt 0.937
(b)	B1	Correct contextual statement
(c)	M1	For use of a correct model ie a correct expression for b
	M1	For use of a correct model ie a correct expression (ft) for a
	A1	For a correct model $w = 15.4 + 6.42f$ with awrt 15.4 and awrt 6.42
(d)	B1ft	awrt 3.21 condone -3.21
(e)	M1	Explaining a reason for their conclusion eg there is **a pattern/trend** in the **residuals** Do not accept residuals not close to zero
	A1	concluding it is not valid oe
(f)	B1	A comment about not using a linear line eg use a quadratic model, logarithmic graph exponential

Question	Scheme	Marks	AOs

3(a)	$H_0: \sigma_A^2 = \sigma_B^2,\ H_1: \sigma_A^2 \neq \sigma_B^2$	B1	2.5
	$s_A^2 = \frac{1}{24}\left(1637.37 - 25\times\left(\frac{194.7}{25}\right)^2\right) = 5.0436$	M1 A1	2.1 1.1b
	$s_B^2 = \frac{1}{25}\left(2031.19 - 26\times\left(\frac{227.5}{26}\right)^2\right) = 1.6226$	A1	1.1b
	$\frac{s_A^2}{s_B^2} = 3.108...$	M1	3.4
	critical values upper tail $F_{24,25} = 1.96$	B1	1.1b
	There is evidence that the two **variances** are different.	A1ft	2.2b
		(7)	
(b)	The **yields** are normally distributed.	B1	1.2
		(1)	
			(8 marks)

Notes:

(a)	**B1**	both hypotheses correct using σ **or** σ^2
	M1	Using a correct method for either s_A^2 or s_B^2. May be implied by a correct value
	A1	awrt 5.04
	A1	awrt 1.62
	M1	Using the F-distribution as the model eg $\frac{s_A^2}{s_B^2}$ $\left(\text{allow } \frac{s_B^2}{s_A^2}[=0.321...]\right)$
	B1	awrt 1.96 or 0.506 must match their method
	A1ft	Drawing a correct inference following through their CV and value for $\frac{s_B^2}{s_A^2}$ or $\frac{s_A^2}{s_B^2}$ Allow $\sigma_B^2 \neq \sigma_A^2$ Allow standard deviation instead of Var . Do not allow $\sigma_B^2 = \sigma_A^2$
(b)	**B1**	recalling the fact that the variable **yield** needs to be normally distributed

Question	Scheme	Marks	AOs
4(a)	$k\left(2^3-\frac{3}{8}2^4\right)=1$	B1*	1.1b
	$2k=1$		
	$k=\frac{1}{2}$ *		
	Or $\frac{1}{2}\left(2^3-\frac{3}{8}2^4\right)=1 \quad \therefore k=\frac{1}{2}$ *	(B1*)	
		(1)	
(b)	$\mathrm{f}(x)=k\left[3x^2-\frac{3}{2}x^3\right]$	M1	2.1
	(i) $\int_0^2 x\mathrm{f}(x)\,\mathrm{d}x=k\int_0^2\left(3x^3-\frac{3}{2}x^4\right)\mathrm{d}x$	M1d	1.1b
	$=\left[\frac{3x^4}{8}-\frac{3x^5}{20}\right]_0^2$		
	$=\frac{6}{5}$ or 1.2	A1	1.1b
	(ii) $3x-\frac{9x^2}{4}=0$	M1d	3.1a
	$x\left(3-\frac{9x}{4}\right)=0$	M1d	1.1b
	$x=0$ or $\frac{4}{3} \qquad \therefore$ mode $=\frac{4}{3}$	A1	1.1b
		(6)	
(c)	Mode > mean implies it is negative skew	B1ft	2.4
		(1)	
			(8 marks)

Notes:

(a)	**B1***	substituting $x = 2$ into F(x) and equating to 1 leading to $k=\frac{1}{2}$ with no errors. Minimum subst seen is $k(8-6)=1$ or $0.5(8-6)=1$
(b)	**M1**	Realising they need to find the pdf and attempting to differentiate $k\left[x^3-\frac{3}{8}x^4\right]$ at least 1 correct term
(i)	**M1d**	dep on 1st M1 Attempting to find $\int_0^2 x(\text{their } \mathrm{f}(x))\,\mathrm{d}x$ At least one correct term ft their pdf
	A1	$\frac{6}{5}$ or 1.2 oe NB 1.2 with no working gains M0M0A0
(ii)	**M1d**	dep on 1st M1 for realising they need to differentiate their pdf. At least one correct term but ft their pdf
	M1d	Dep on 3rd M1. correct method for solving their differential of their pdf = 0 pdf must be of the form ax^2+bx
	A1	$\therefore$ mode $=\frac{4}{3}$ only. They must eliminate 0
(c)	**B1ft**	ft their mode and mean or a correct sketch.

Question	Scheme	Marks	AOs
5	d: 6 –6 12 6 –4 1 7 14	M1	3.1b
	$\bar{d} = \pm 4.5$ $\quad s_d = \sqrt{50.285...} = 7.09 \ldots$	M1	1.1b
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d \neq 0$	B1	3.3
	$t = \pm\dfrac{"4.5"\sqrt{8}}{"7.09..."}$ oe	M1	1.1b
	$= \pm\ 1.7948 \ldots.$ awrt $\pm 1.79/1.8$	A1	1.1b
	Critical value $t_7 = \pm 3.499$	B1	1.1b
	There is insufficient evidence that the **papers** are of a different level of difficulty or **Alexa's** belief is correct	A1ft	2.2b
		(7)	
			(7 marks)

Notes:

M1: for realising that the model to use is the paired t-test and finding the differences $(\pm)$ At least 3 correct

M1: correct method for finding $\bar{d}$ and s_d .

B1: Using a correct model for difference and both hypotheses correct using the notation μ_d or μ
Condone $\mu_I = \mu_{II}$ and $\mu_I \neq \mu_{II}$

M1: Using the correct method to find test statistics ie $t = \pm\dfrac{"\text{their } 4.5"\sqrt{8}}{"\text{their } 7.09..."}$

A1: awrt1.79 or 1.8

B1: for correct critical value $t = \pm 3.499$ with compatible sign

A1ft: Drawing a correct inference in context using their CV and their value of t

NB difference of means test gets M0M0B1M0A0B0A0

Question	Scheme	Marks	AOs
6	99% confidence interval for Var uses χ^2 values of 1.735 or 23.589	B1	3.3
	$\frac{9s^2}{1.735}=0.2328$ or $\frac{9s^2}{23.589}=0.01712$	M1	2.1
	$s^2=\frac{0.2328\times\text{"}1.735\text{"}}{9}$ or $\frac{0.01712\times\text{"}23.589\text{"}}{9}$ $[=0.04487...]$	dM1	1.1b
	$\bar{x}=4.84$	B1	1.1b
	$\text{H}_0:\mu=5$ $\text{H}_1:\mu<5$	B1	2.5
	CV $t_9=-1.833$	B1	1.1b
	$t=\pm\frac{\text{"}4.84\text{"}-5}{\sqrt{\text{"}0.0449\text{"}/10}}$	M1	1.1b
	$=$ awrt -2.39	A1	1.1b
	Stan's belief is supported **or** there is evidence that the **mean diameter** of the bolts is less than **5mm**	A1ft	2.2b
		(9)	

(9 marks)

Notes:

B1: For realising a χ^2 distribution must be used as a model and finding a correct value

M1: For realising the need to set $\frac{9s^2}{\text{"smallest }\chi^2\text{"}}=0.2328$ or $\frac{9s^2}{\text{"largest }\chi^2\text{"}}=0.01712$

dM1: correct method used to solve equation to find s^2

B1: awrt 4.84

B1: Both hypotheses correct using the notation μ

B1: $\pm$ 1.833

M1: For us of correct formula ie $\pm\frac{\text{"their 4.84"}-5}{\sqrt{\text{"their 0.0449"}/10}}$ If "4.84" not shown it must be correct here

A1: -2.39

A1ft: Drawing a correct inference following through their CV and test statistic (must have matching signs)

NB if chi squared values not shown
$s^2=0.045$ or 0.0449 award B0 M1M1 for awrt 0.04487 award B1 M1 A1

Use of $2(2.5758)\frac{\sigma}{\sqrt{10}}=0.21568$ gives $\sigma=\sqrt{0.0175}$ could get B0M0M0B1B1B1M0A0A0

Unless continue to get $s^2=\frac{10}{9}0.0175=0.0194...$

Use of $2(1.833)\frac{s}{\sqrt{10}}=0.21568$ gives $s=0.1860$ could get B0M0M0B1B1B1M1A0A1

Question	Scheme	Marks	AOs
7(a)	Let $T = W - 2X$ then $\text{E}(T) = 2.5 - 2\times1.27$	M1	3.3
	$= -0.04$	A1	1.1b
	$\text{Var}(T) = 0.7^2 + 2^2 \times 0.4^2$	M1	2.1
	$= 1.13$	A1	1.1b
	$\text{P}\left(Z > \frac{0 - \text{"}-0.04\text{"}}{\sqrt{\text{"}1.13\text{"}}}\right) = \text{P}(Z > 0.0376\ldots)$	M1	2.1
	= awrt 0.484/0.485	A1	1.1b
		(6)	
(b)	$B = W_1 + W_2 + \ldots + W_n + X_1 + X_2 + \ldots + X_{2n}$	M1	3.3
	$\text{E}(B) = 5.04n$	B1	1.1b
	$\text{Var}(B) = n\times0.7^2 + 2n\times0.4^2$		
	$= 0.81n$	A1	1.1b
	$\pm\frac{252 - \text{"}5.04n\text{"}}{\sqrt{\text{"}0.81n\text{"}}}$	M1	1.1b
	$\frac{252 - \text{"}5.04n\text{"}}{\sqrt{\text{"}0.81n\text{"}}} = 0.8$	M1	2.1
	$5.04n + 0.72\sqrt{n} - 252 = 0$ oe		
	$\sqrt{n} = -7.14...$ or 7	M1	1.1b
	$n = 7^2$	M1	1.1b
	$= 49$	A1cso	1.1b
		(8)	
			(14 marks)

Notes:

(a) M1: selecting and using an appropriate model. ie $\pm(W - 2X)$ May be implied by –0.04

A1: –0.04 oe

M1: for realising the need to use Var(W) + 4 Var(X). Allow use of 0.7 for Var (W) instead of 0.7^2 and/or 0.4 for Var(X) instead of 0.4^2. May be implied by 1.13

A1: 1.13 only

M1: For realising the P(T > 0) is required and an attempt to find it. $\frac{0 - \text{"their } -0.04\text{"}}{\sqrt{\text{"their } 1.13\text{"}}}$ may be implied by a correct answer. If E(T) and Var(T) have not been given they must be correct here

A1: awrt 0.484/0.485

(b)M1: Selecting and using appropriate model. May be implied by 0.81

B1: $5.04n$ only

A1: $0.81n$

M1: For standardising using their mean and sd $\pm\frac{252 - \text{"}5.04n\text{"}}{\sqrt{\text{"}0.81n\text{"}}}$ If mean and sd not given they must be correct here

M1: For constructing an equation and equate their standardisation to 0.8 or awrt 0.7998. Must be of form $\frac{252 - an}{b\sqrt{n}} = 0.8$ or $\frac{252 - an}{bn} = 0.8$

M1: Correctly solving their 3 term quadratic equation. Condone $n = 7$

M1: for realising the need to square their answer or for attempting to square their quadratic equation

A1cso: 49 only

Question	Scheme	Marks	AOs
8(a)	$H_0: \rho_s = 0 \quad H_1: \rho_s > 0$	B1	2.5
	CV = 0.6	B1	1.1b
	$r_s = 0.85$ does lie in the critical region	M1	2.1
	There is evidence to suggest that there is a relationship between the position in the 100m sprint and the position in the long jump.	A1	2.2b
		(4)	
(b)	$1 - \dfrac{6\sum d^2}{9(80)} = 0.85$	M1	3.1b
	$\sum d^2 = 18$	A1	1.1b
	$\sum d^2$ needed is $'18' - 15 = 3$	M1	1.1b
	Since $\sum d^2 = 3$ for the 3 missing places each place must contribute 1, therefore *B* must be in position 5 or 7. However, 5 has already been used so they must be position 7	A1	2.2a
	C is 6th and *D* is 8th	A1	2.2a
	SC B7, C6, D8 with no reasons B1 marks as final A1 on epen		
		(5)	
(c)	The $\sum d^2$ will not change but the value of n will decrease therefore	M1	2.4
	Spearman's rank correlation will decrease	A1	2.2a
		(2)	
	Notes:	**(11 marks)**	

(a)B1: Both hypotheses correct written using the notation ρ

B1: awrt 0.6

M1: Drawing a correct inference using their CV and the value of r_s

A1: Drawing a correct inference in context using their CV and the value of r_s

(b)M1: For realising they need to equate $1 - \dfrac{6\sum d^2}{9(80)}$ to 0.85 to enable them to find the $\sum d^2$

A1: 18

M1: for $\sum d^2 = 3$

A1: For using the information in the question with the value for $\sum d^2$ to deduce that each must contribute 1 to the $\sum d^2$ and explain why *B* must be in position 7

A1: C 6th D 8th

(c)M1: Complete explanation why it decreases

A1: using the information given to deduce that it decreases

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